University of Tripoli - Faculty of Engineering
Department of Electrical and Electronics Engineering

EE302 Signals and Systems

Final Exam Solution, Fall 2017, 04 February 2018

Answer the following Questions

Q1)

[2] **i)** Determine the correct type of each signal, justify your answer.

Signal	Energy signals, power signals or neither?	Why?
$x(t) = \cos\left(\frac{\pi}{3}t - \frac{\pi}{4}\right)$	Power signal	Magnitude of x(t) $\rightarrow \infty$ as $t \rightarrow \infty$
$x[k] = (-0.2)^k u[k]$	Energy Signal	Magnitude of x[k] $\rightarrow 0$ as $k \rightarrow \infty$

[3] **ii)** Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period and the harmonics present in x(t).

Signal	Periodic?	ω_0	Harmonics present
$\cos\left(\frac{\pi}{3}t - \frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}t\right)$	yes	$\frac{\pi}{3}$	ω_0 & $2\,\omega_0$
$\cos\left(\frac{1}{5}k\right) + \cos\left(\frac{1}{4}k\right) + \cos(\frac{1}{2}k)$	yes	$\frac{1}{20}$	$4\,\omega_0, 5\,\omega_0$ & $10\,\omega_0$

[3] **iii)** Determine the properties of each systems (Yes or No)

System	Linear?	Casual?	Time-Invariant?	Invertible?	inverse system
$y(t) = 2x^2(t)$	No	yes	yes	No	-
y[k]=k x[k]	yes	yes	No	No	-
y[k] = 8 x[k]	yes	yes	yes	yes	1/8

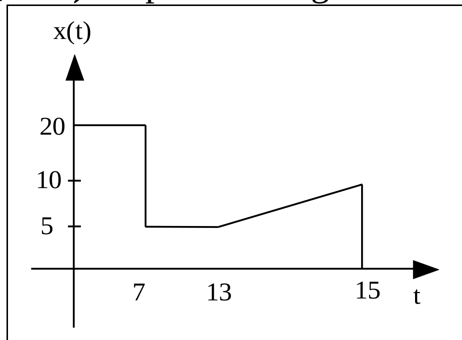
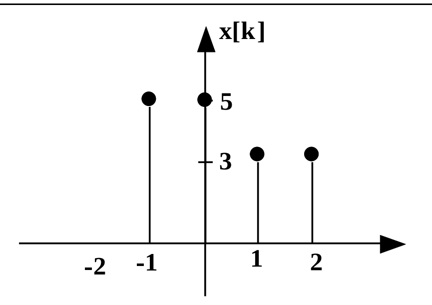
[3] **iv)** Evaluate the following integrals

$\int_1^2 (2t - 1)\,\delta(t)dt$	0
$\int_{-2}^2 exp(2t)\,\delta(t-1)dt$	$exp(2) = 7.3891$

[3] **v)** Find the impulse response of the given discrete system

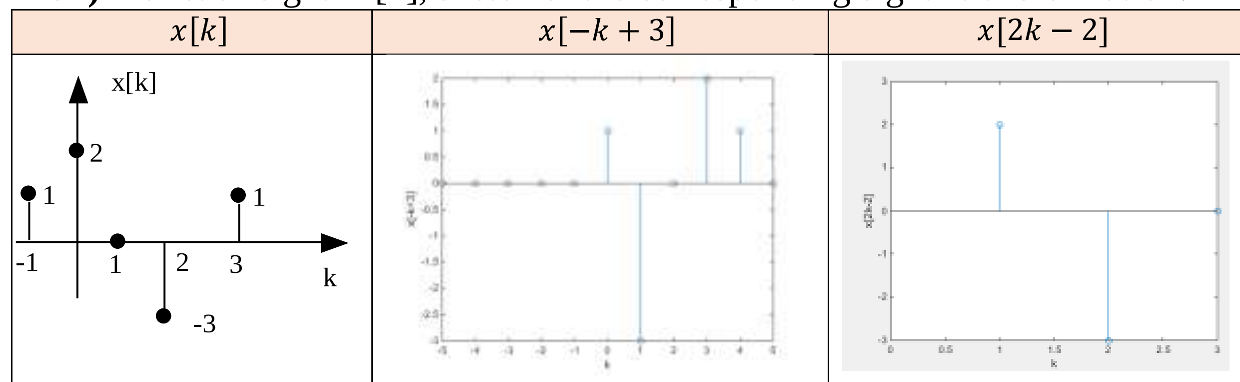
	impulse response
x[k] → D → 1/3 → D → D → Σ → y[k]; branches: 1/5 (−), 3 (+)	$y[k] = 3x[k] - \frac{1}{15}x[k-2] + \frac{1}{3}x[k-3] = 0$ $h[k] = \left\{3,0,-\frac{1}{15},\frac{1}{3}\right\}$

[3] **vi)** Express the signals shown in terms of unit step functions

	$x(t) = 20u(t) - 15u(t-7) - 5u(t-13) + \left(2.5 * t - \frac{55}{2}\right) * (u(t-13) - u(t-15))$
	$x[k] = 5u[k+1] - 2u[k-1] - 3u[k-3]$

[4] **vii)** For each signal x[k], sketch of the corresponding signal transformation.

$x[k]$	$x[-k+3]$	$x[2k-2]$



[3] **viii)** Find the Fourier Transform of the following signals.

$3\cos(30t-2) + 2\cos(50t+2)$	$3\pi\left[\delta(\omega-30)e^{-2j} + \delta(\omega+30)e^{2j}\right] + 2\pi\left[\delta(\omega-50)e^{2j} + \delta(\omega+50)e^{-2j}\right]$
$3 + 4\delta(t+4) - 8\delta(t-3)$	$6\pi\,\delta(\omega) + 4e^{j4\omega} - 8e^{-j3\omega}$

[3] **ix)** Find the Laplace Transform of the following signals.

$2(t-5)u(t-5)$	$\frac{2}{s^2}e^{-5s}$
$3u(t-1) + \delta(t-1)$	$\frac{3}{s}e^{-s} + e^{-s}$

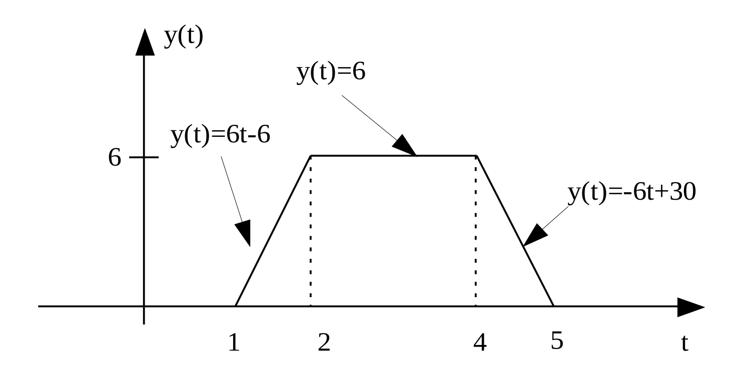
[3] **x)** Determine if the following systems stable or unstable, justify your answer

	Stable?	Why?
$y''(t) - 1.5y'(t) + y(t) = 0$	No	$\lambda_{1,2}=0.75\pm j0.66$ Poles of the system in the RHP
$y[k+2] - 0.5y[k+1] + y[k] = 0$	Marginally stable	$\gamma_{1,2}=0.25\pm j0.96$, $\lvert\gamma_{1,2}\rvert=1$

Q2 –

$$y(t) = x(t) * h(t) = \begin{cases} 6t - 6 & 1 \le t \le 2 \\ 6 & 2 \le t \le 4 \\ -6t + 30 & 4 \le t \le 5 \end{cases}$$

$$y(t) = 6(t-1)u(t-1) - 6(t-2)u(t-2) - 6(t-4)u(t-4) + 6(t-5)u(t-5)$$



Q3 –

$$Y(s) = \frac{X(s)}{Z(s)} = \frac{X(s)}{2s+4}$$

a)

$$x(t) = 5\delta(s) \Leftrightarrow X(s) = 5$$

$$Y(s) = \frac{5/2}{s+2} \Leftrightarrow y(t) = 2.5\, e^{-2t}\, u(t)$$

a)

$$x(t) = 3e^{-5t}u(t) \Leftrightarrow X(s) = \frac{3}{s+5}$$

$$Y(s) = \frac{0.5}{s+2}\frac{3}{s+5} = \frac{0.5}{s+2} - \frac{0.5}{s+5}$$

$$y(t) = 0.5[\, e^{-2t} - \, e^{-5t}]\, u(t)$$

Q4 –

a)

$$x(t) = cos(2t)\sin(3t) = \frac{1}{2}[\sin(t) + \sin(5t)] = \frac{-j}{4}(e^{it} - e^{-it} + e^{i5t} - e^{-i5t})$$

$$b_1 = b_5 = \frac{1}{2} \qquad D_1 = D_{-1} = \frac{-jb_1}{2} = \frac{-j}{4} \qquad D_5 = D_{-5} = \frac{-jb_5}{2} = \frac{-j}{4}$$

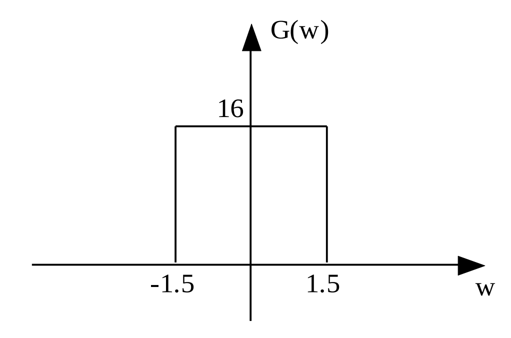
$$D_n = 0 \;\; otherwise$$

b)

$$y(t) = \sin(5t)$$

Q5 –

$$x(t) = g(t)\cos(4.5t) \Leftrightarrow \mathrm{X}(\omega) = \frac{1}{2}[G(\omega - 4.5) + G(\omega + 4.5)]$$



$$rect\left(\frac{t}{\tau}\right) \Leftrightarrow \tau \,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

Use the duality property

$$\tau \,\mathrm{sinc}\left(\frac{\tau t}{2}\right) \Leftrightarrow 2\pi\, rect\left(\frac{\omega}{\tau}\right) \quad \rightarrow \quad 3\,\mathrm{sinc}\left(\frac{3\mathrm{t}}{2}\right) \Leftrightarrow 2\pi\, rect\left(\frac{\omega}{3}\right)$$

$$\mathrm{x(t)} = \frac{16\times 3}{2\pi}\,\mathrm{sinc}\left(\frac{3\mathrm{t}}{2}\right)\cos(4.5t)$$